dynamics of queues. One of the most powerful application of averages is contained in a simple mathematical relation known as Little's law. Little's law appears in many guises throughout the literature on performance analysis of both computer systems and manufacturing systems.

In queueing theory notation, Little's law:

$$Q_k = \lambda R_k \,, \tag{2.14}$$

states that the average queue length Q is equal to the average arrival rate λ into the queueing center defined by (2.1) multiplied by the time spent at the queueing center, i.e., the average residence time R defined by (2.12). Applying the definitions of the utilization (2.10) and arrival rate (2.1), we can also write:

$$\rho_k = \frac{C_k}{T} \times \frac{B_k}{C_k} = \lambda S_k \;. \tag{2.15}$$

This is a special case of (2.14) where the waiting time prior to receiving service is not included.

2.5.1 A Little Intuition

Equation (2.14) can be appreciated intuitively with the aid of the following example. It is the "rush hour" commute and you are stuck in traffic at the entrance to a toll way. At this time of day it takes 15 min to get past the toll booths and onto the freeway. While waiting, you start counting cars arriving at the toll-way entrance during a 5 min interval. You see 25 new cars in 5 min. How many cars are waiting with you at the toll entrance? Little's law tells us how to estimate that number.

We know the arrival rate λ is 25 new cars in a 5 min period, and we know that it takes 15 min to get through the toll entrance, which is our residence time R. Applying (2.14) produces:

$$\frac{25 \text{ new cars}}{5 \text{ min}} \times 15 \text{ min} = 75 \text{ cars} \,. \tag{2.16}$$

Here, $Q = 25$ cars refers to the total number of cars enqueued at the toll entrance. It is an average value because there may actually be more or fewer cars, statistically speaking. If a sufficient number of measurements are repeated, however, they should converge to the value predicted by Little's law.

Seen in this way, Little's law may not appear all that remarkable. But remember that queues, like a toll-way entrance or the line at the grocery checkout, are subject to significant fluctuations over short intervals of time. The exact number of cars or customers enqueued at any instant cannot be known ahead of time, it can only be expressed as a probability estimate. Even in the presence of fluctuations, however, an essential feature of queueing can be expressed in terms of the average quantities in Little's law. This result is actually more general than queueing theory and J. D. Little [1961] (see `www.`

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